Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2013

Question/Answer Booklet

MATHEMATICS 3C/3D

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 13-088
MAT3CD-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	11	11	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 **(7 marks)**

Alex finishes work between 5 pm and 6 pm every weekday. His finishing time T, in minutes after 5 pm, is a uniformly distributed random variable:

$$T \sim U(0, 60).$$

(a) What is the probability that Alex will finish work after 5.15 pm? (1 mark)

(b) Determine

(i) the mean of T. (1 mark)

(ii) $P(T = 55)$. (1 mark)

(iii) $P(T > 55 \mid T > 40)$. (2 marks)

(iv) the value of t for which $P(T > t) = P(T < 2t)$. (2 marks)

Question 9 **(12 marks)**

A mining company has two sources of gold ore: mine A and mine B.

- 2 grams of gold can be extracted from each tonne of ore from mine A.
- 3 grams of gold can be extracted from each tonne of ore from mine B.

Ore from both sources is processed at a single processing plant.

- In order to keep the plant running, a total of at least 3 tonnes of ore must be processed each hour.
- Staffing constraints at the mines determine that the amount of ore processed from mine B cannot exceed twice the amount of ore processed from mine A.
- Ore from mine A costs \$20 per tonne to process, and ore from mine B costs \$10 per tonne to process. Processing costs must be kept to no more than \$80 per hour.

Let x = the number of tonnes of ore per hour processed from mine A,

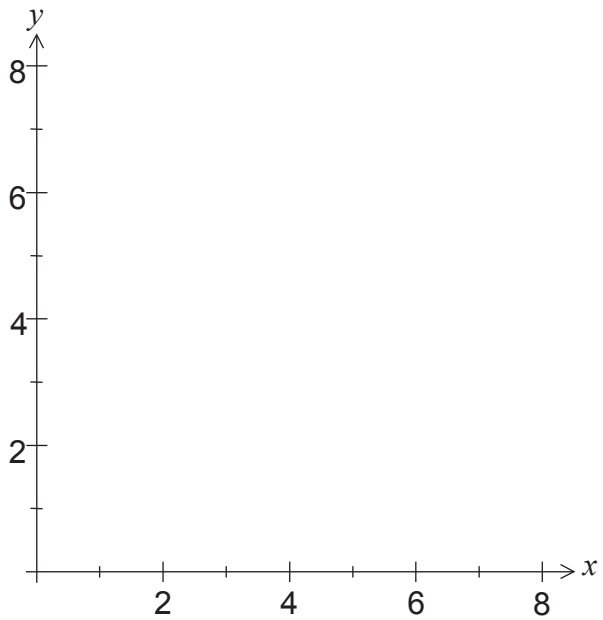
and y = the number of tonnes of ore per hour processed from mine B.

The following four constraints can be obtained from the information above.

$$x \geq 0 \qquad y \geq 0 \qquad x + y \geq 3 \qquad 20x + 10y \leq 80$$

(a) State the final constraint that applies to this situation. (2 marks)

(b) Draw all the constraints on the axes below and indicate the feasible region. (4 marks)



See next page

(c) The company wants to maximise the total weight of gold extracted. How many tonnes of ore from each mine should be processed each hour? (4 marks)

(d) A new technique enables k grams of gold to be extracted from each tonne of ore from mine A, where $k > 2$.

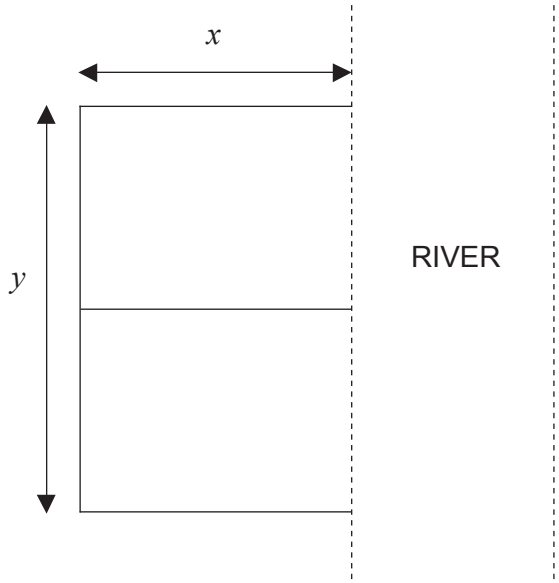
What is the smallest value of k that would enable the maximum weight of gold to be extracted without processing any ore from mine B? (2 marks)

See next page

Question 10 **(7 marks)**

A farmer has $1500 available to build an E-shaped fence along a straight river so as to create two identical rectangular pastures.

The materials for the side parallel to the river cost $6 per metre and the materials for the three sides perpendicular to the river cost $5 per metre.



Each of the sides perpendicular to the river is x metres long, and the side parallel to the river is y metres long.

(a) Assuming that the farmer spends the entire $1500, show that the total area $A(x)$ of the two pastures, in square metres, is $A(x) = \frac{5}{2}(100x - x^2)$. (3 marks)

See next page

(b) Use calculus methods to determine the dimensions of the fence that maximise the total area, and state this area. (4 marks)

See next page

Question 11 **(13 marks)**

The size of a population of birds is changing according to the rule $\frac{dP}{dt} = -0.08P$,

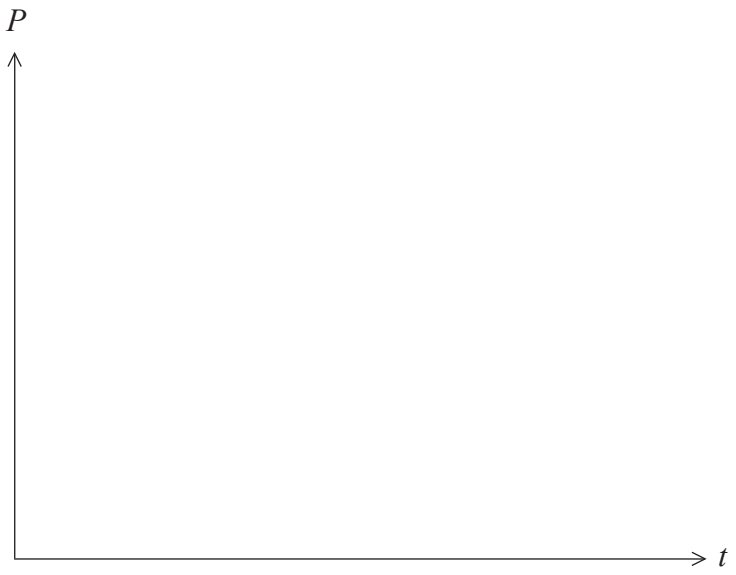
where P is the number of birds in the population, and t is the time in years from the initial population measurement.

There are initially 1000 birds in the population.

(a) Describe the type of relationship between P and t. (2 marks)

(b) State an equation for P in terms of t. (1 mark)

(c) Sketch the graph of P against t on the axes below. (3 marks)



(d) Determine

(i) the number of birds in the population after 10 years. (1 mark)

(ii) the number of years, to one decimal place, after which there are 800 birds in the population. (3 marks)

(e) What is the value of $\frac{dP}{dt}$ when $t = 10$?

Interpret this answer in terms of the bird population. (3 marks)

See next page

Question 12 **(13 marks)**

The length of barramundi is approximately normally distributed with a mean of 650 mm and a standard deviation of 100 mm. For game fishing, a barramundi must be between 550 mm and 800 mm long to be considered of legal size.

(a) What is the probability that a randomly caught barramundi is of legal size? (1 mark)

(b) A fisherman catches 100 barramundi in a week. What is the expected number of legal-sized fish in his catch? (1 mark)

(c) What is the probability that a legal-sized barramundi is over 750 mm in length? (2 marks)

(d) Calculate the interquartile range of the barramundi population. (3 marks)

See next page

(e) A fisheries researcher suspects that the length of the barramundi population may have changed over time. She intends to investigate this by taking random samples of barramundi and calculating the mean length. Assume that the standard deviation of the fish population is still 100 mm.

(i) Her first sample of 50 barramundi had a mean length of 668 mm. Use this to calculate a 90% confidence interval for the mean length of the population, and explain whether this provides strong evidence that the population mean had changed from 650 mm. (4 marks)

(ii) With her second sample, she wants to obtain a 95% confidence interval for the mean length of the barramundi population which has a width of no more than 20 mm. What sample size should she select? (2 marks)

Question 13 **(9 marks)**

For two events, A and B, $P(A) = 0.4$ and $P\left(\overline{A \cup B}\right) = 0.2$.
Let $P(A \cap B) = x$.

(a) Determine an expression for $P(B)$ in terms of x. (2 marks)

(b) Determine the value of x under each of the following conditions.

(i) A and B are mutually exclusive events. (1 mark)

(ii) $P(B \mid A) = 0.25$. (2 marks)

See next page

(iii) $P(A \mid B) = \frac{1}{3}$. (2 marks)

(iv) A and B are independent events. (2 marks)

See next page

Question 14 **(7 marks)**

A computer store room contains 25 computers. Of these, 20 are working and five need to be repaired. An order for five computers is received. The computer technician selects five computers at random from the store room.

(a) What is the probability that all five are working? (2 marks)

(b) What is the probability that more than one needs to be repaired? (3 marks)

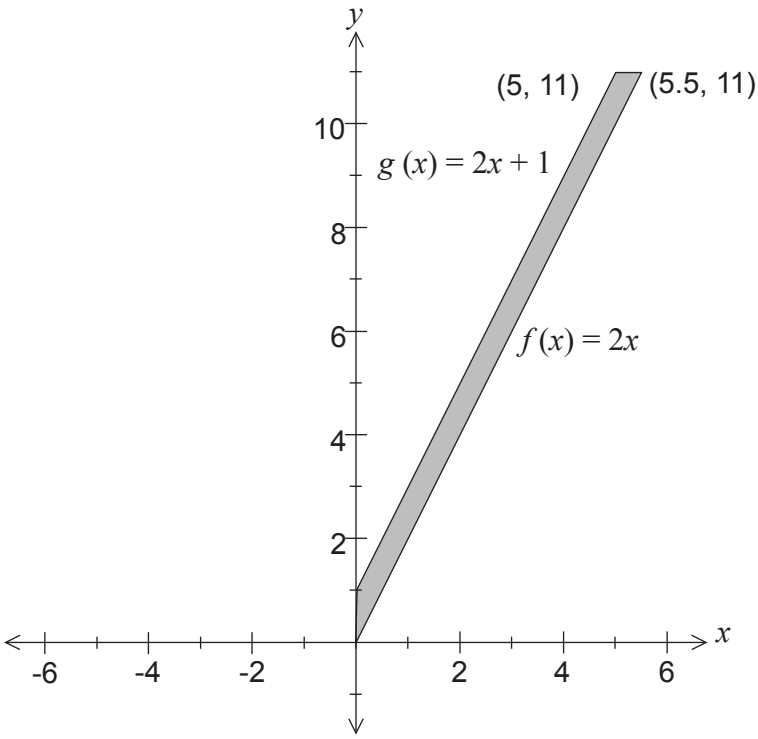
(c) The technician checks the first three of the five computers and finds that they are all working. What is the probability that all five are working? (2 marks)

See next page

Question 15 **(5 marks)**

A conical container is made by rotating the shaded area shown in the graph around the y -axis.

The area is bounded by the lines $x = 0, y = 11, f(x) = 2x$ and $g(x) = 2x + 1$.



Use calculus methods to determine the volume of material required to construct the container.

See next page

Question 16 **(11 marks)**

A factory makes low-cost batteries. Unfortunately, 20% of the batteries are faulty.

Batteries are sold in packets of 20.

(a) Let X be the number of faulty batteries in a packet.

(i) State the distribution of X and determine its mean and standard deviation. (3 marks)

(ii) Calculate $P(X \geq 5)$. (1 mark)

(b) A customer buys 20 packets of batteries.
Let Y be the mean number of faulty batteries per packet.
According to the Central Limit Theorem, Y will be approximately normally distributed.

(i) Determine the mean and standard deviation of Y. (2 marks)

(ii) Calculate $P(Y \geq 4.2)$. (1 mark)

See next page

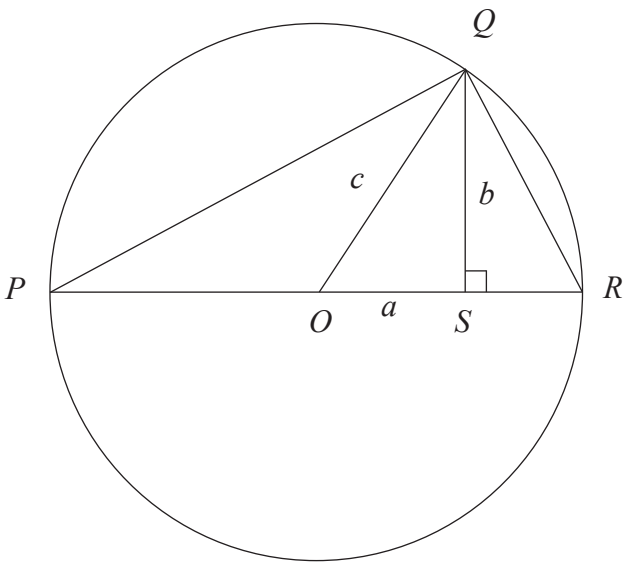
(c) In a sample of 400 batteries, what is the probability that at least 84 will be faulty? (2 marks)

(d) Explain why the answer in part (c) should be close to the answer in part (b) (ii). (1 mark)

(e) Explain why the answer in part (c) should **not** be exactly equal to the answer in part (b) (ii). (1 mark)

See next page

Question 17 **(7 marks)**



The circle above has centre O. PR is a diameter of the circle. QS is drawn perpendicular to PR.

In triangle OSQ, let $OQ = c$, $OS = a$ and $SQ = b$.

(a) Prove that triangle PSQ is similar to triangle QSR. (4 marks)

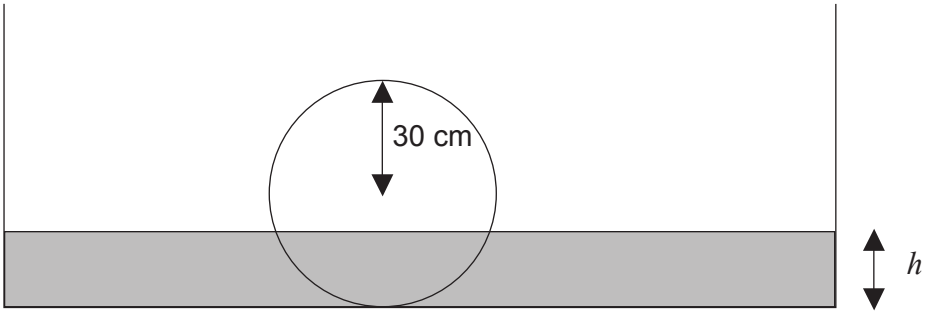
(b) Use the result from part (a) to prove the Pythagorean theorem $c^2 = a^2 + b^2$. (3 marks)

See next page

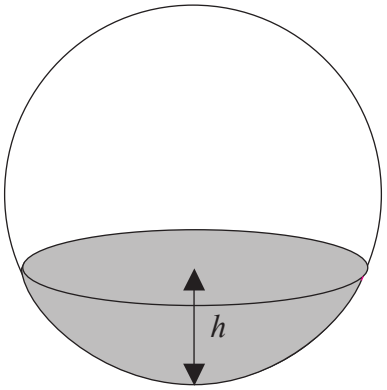
Question 18 **(9 marks)**

A sphere of radius 30 cm is sitting on the base of a container of water. The water level is rising at a rate of 2 cm per second, so that the sphere is gradually becoming submerged.

Let h be the height of the water.



The submerged portion of the sphere is as shown below.



The volume of the submerged portion is given by

$V = \frac{\pi h^2}{3}(3r - h),\ 0 \leq h \leq 2r$, where r is the radius of the sphere.

(a) Use this formula to calculate the volume of the entire sphere, to the nearest cubic centimetre. (1 mark)

See next page

(b) Find the values of h for which $\frac{dV}{dh} = 0$. (3 marks)

(c) Calculate the rate at which the submerged volume of the sphere is changing at the time when the sphere is half submerged. (3 marks)

(d) Consider the following conjecture:
Every 1% increase in h leads to approximately a 3% increase in V.
Using the formula $\delta V \approx \frac{dV}{dh}\delta h$, explain whether the conjecture is true or false. (2 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Published by the School Curriculum and Standards Authority of Western Australia
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